Effect of Transverse Curvature on Turbulent Boundary-Layer Mass Transfer

S. S. CHEN

Tufts University, Medford, Massachusetts

Axisymmetrical boundary layers occur in flows past axially symmetrical bodies. They differ from the two-dimensional case by including the effect of curvature in a plane normal to the flow direction. In general, a transverse curvature decreases the rates of heat and mass transfer in the case of flow in a pipe (concave curvature), whereas for flow outside an external surface, a transverse curvature (convex curvature) increases the rate of heat and mass transfer. This can be physically understood, since in the former case the surface area normal to the heat or mass flux decreases as it moves away from the solid-fluid interface, and the situation is reversed in the latter case. The effect of transverse curvature on skin friction has been studied extensively (*1* to *5*). The results consistently indicate that skin friction increases with increasing convex transverse curvature for both laminar and turbulent flows. The analysis of Seban and Bond (*2*) for the laminar boundary layer along a cylinder also arrived at the conclusion that heat transfer coefficient increased with increasing curvature. No satifactory calculations have been made on the heat or mass transfer in turbulent axisymmetrical boundary layer.

The purpose of this investigation was to study experimentally the rate of mass transfer from a 1-in. diameter porous cylinder to an air stream. The results were compared with theoretical predictions and experimental results for flow along a flat plate. It was found that the transverse curvature significantly increased the mass transfer rates.

EXPERIMENTAL METHOD

The principal parts of the equipment in this study were a 1-in.-diameter porous cylinder, a liquid injection system, and an air supply system.

The porous cylinder with a hemispherical nose piece was placed vertically above a converging square air jet, which has an opening of 8 $\times$ 8 in. The smoothly converging jet succeeded in providing a flat velocity profile with a maximum deviation of 1% up to 20 in. above the jet opening. The level of turbulance of air stream leaving the jet was approximately 1.3%. The temperature of the air stream was controlled by wire grid heaters in the air duct to within 0.1°F. of the desired value. The air velocity ranged from 4 to 35 ft./sec.

A schematic diagram of the test cylinder is shown in Figure 1. The porous cylinder consisted of an inert approach section, a porous section, and a downstream section. The porous section was made of diatomacious earth. Four approach sections of different lengths from 2 to 20 in. were used. By using different approach lengths, the local Reynolds number could be varied independent of the air velocity, and the influence of the approach length on the rate of mass transfer could also be studied. Two porous sections of 0.500 and 0.715 in. were tested. Isothermal mass transfer was achieved by supplying energy from a heater coil placed in helical grooves beneath the outer surface of the porous sections. The downstream section had an inner passage of 1/8-in. diameter which served as a liquid supply line to the porous section. The thermocouple and heater leads were also passed through this passage.

The evaporating component was supplied to the evaporating surface at a predetermined rate from a liquid injector. The liquid rate was so controlled that the evaporating surface was completely covered by a very thin liquid film, but no excess liquid was accumulated on the surface. The surface temperatures were measured with thermocouples made of 0.003-in. copper and constantan wires passed from the inside of the cylinder and extended to the surface at an angle of 45 deg. The wires traveled an equal distance on the surface to meet at the junction. A groove had been cut beneath the wires so that about one half of the wire diameter was under the surface. In addition, a traveling thermocouple was used to measure the surface temperature at different positions in the flow direction. The readings from the traveling thermocouple and fixed thermocouples agreed within 0.1°F.

Runs were made both for isothermal evaporation by supplying energy to the heater beneath the surface and for non-isothermal evaporation when no energy was supplied to the liquid. In the former case, a reasonable uniform surface temperature was achieved by varying the pitch of heating coil to supply more energy to the lower side of the porous section for higher local evaporation rates. In the latter case, the surface temperatures were 10 to 20 deg. lower than the air temperature. However, the results of the two cases agreed well within experimental errors when corrections for temperature were made.

Heptane and octane were chosen as the evaporating substances.

RESULTS AND DISCUSSIONS

The rate of evaporation was expressed in terms of the Sherwood number defined as

$$N_{Sh,L}=\frac{m_k b_k T L}{2\pi a x_w D_k P \ln\left(p_{j\infty}/p_{js}\right)} \tag{1}$$

The logrithmic partial vapor pressure difference takes into account the finite normal velocity on the surface. A comparison of the data for *n*-heptane and *n*-octane clearly indicated that good agreement was obtained only if the finite normal velocity on the surface was taken into consideration by using Equation (1).

From the plots of $N_{Sh,L}$ vs. $N_{Re,L}$ it was found that the slope for each series of data taken at a fixed value of X_0/L varied from 0.76 to 0.8 for $N_{Re,L} > 15{,}000$. The slight deviations from the well-known relation for turbulent boundary layers

$$N_{Sh,L} \sim N_{Re,L}{}^{0.8} \tag{2}$$

were probably partially due to the influence of the transverse curvature. This is reasonable, since at low veloci-

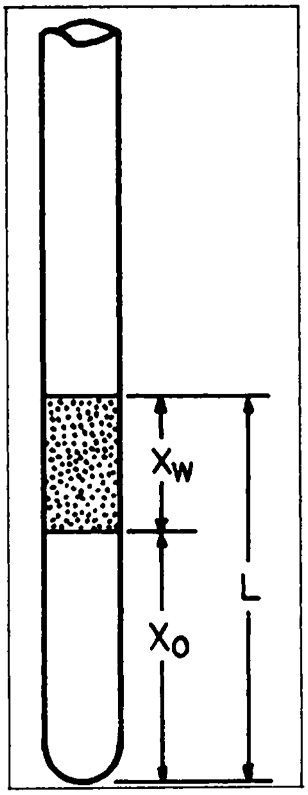

Fig. 1. Schematic diagram of the test cylinder.

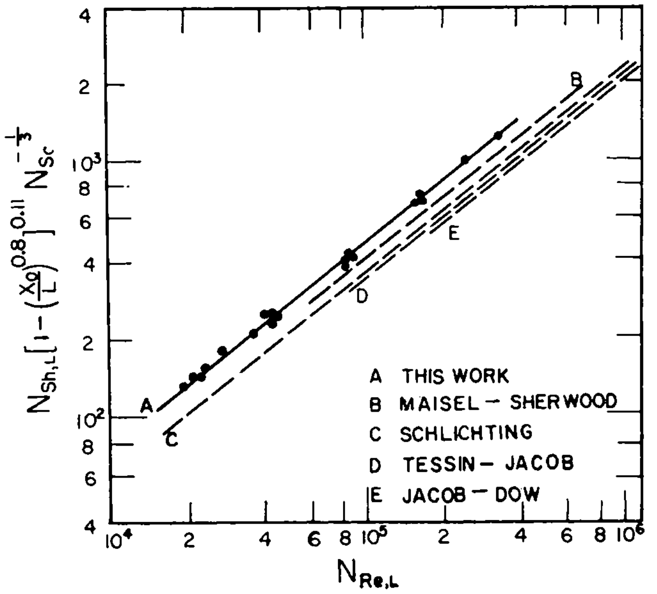

Fig. 2. Comparison of results with previous measurements for turbulent boundary layer.

ties the boundary-layer thickness is greater, and the influence of transverse curvature on the mass transfer rate is accordingly greater. In other words, one expects higher mass transfer rates at low velocities than the predictions from Equation (2).

In the presence of an approach length, the diffusion boundary layer does not begin at the same position as the momentum boundary layer. If x is close to x_0, the presence of this section will noticeably increase the rate of mass transfer from the wetted surface to the air stream owing to the large concentration gradient. Several approach-length functions were tested. It was found that the following formula

$$\frac{N_{Sh,L}}{(N_{Sh,L})_{x0=0}} = [1 - (x_0/L)^{0.8}]^{-0.11} \qquad (3)$$

obtained by Maisel and Sherwood from their mass transfer data for the evaporation of water from a flat plate gave a good correlation of the present data.

Combining Equations (2) and (3) and assuming that the Sherwood number is proportional to $N_{Sc}{}^{1/3}$ and fitting the formula to the experimental data, we arrived at the following formula:

$$N_{Sh,L} = 0.048\, N_{Sc}{}^{1/3}\, N_{Re,L}{}^{0.8}\, [1 - (x_0/L)^{0.8}]^{-0.11} \qquad (4)$$

Experimental results and Equation (4) were plotted in Figure 2 and compared with the theoretical expression of Schlichting (*1*) for heat transfer from a flat plate at zero incidence, Maisel and Sherwood's measurement (*6*) for the evaporation of water from a flat plate, Jacob and Dow's measurements (*7*) for heat transfer from a 1.3-in. cylinder to an air jet, and Tessin and Jacob's measurements (*8*) for heat transfers from a 0.624-in. cylinder. All of the above results were based on the assumption that the Sherwood number (or Nusselt number) was proportional to the one third power of the Schmidt number (or Prandtl number).

The results of Jacob and Dow are lower than results for a flat plate. This is rather difficult to understand, since the convex transverse curvature can only increase the rate of heat transfer from the wall to the fluid stream.

The comparison shows that the results of this work are 15% higher than the data of Maisel and Sherwood and 30% higher than the theoretical prediction of Schlichtip. The higher mass transfer rates obtained in this work shows that the influence of transverse curvature is indeed significant.

NOTATION

a = radius of cylinder
b_k = specific gas constant of the diffusing component
D_k = Fick diffusivity of component k
L = total length from leading edge of cylinder to the downstream edge of wetted section
m_k = tatal material transfer rate of component k
P_j = partial pressure of air
$N_{Re,L}$ = Reynolds number
N_{Sc} = Schmidt number
$N_{Sh,L}$ = Sherwood number, Equation (1)
x_0 = equivalent length of the inert approach section
x_w = wetted length, $L - x_0$
v = kinematic viscosity

Subscripts

j = air
k = diffusing component
s = evaporating surface
∞ = free stream

LITERATURE CITED

1. Schlichting, Hermann, "Boundary Layer Theory," 6 ed., McGraw-Hill, New York (1968).
2. Seban, R. A., and R. Bond, *J. Aeron. Sci.*, **18**, 617 (1951).
3. Glassert, M. B., and M. J. Lighthill, *Proc. Roy. Soc. London*, **A230**, 188 (1955).
4. Stewartson, K., *Quart. Appl. Math.*, **13**, 113 (1955).
5. Young, A. D., *ARC RM 1874* (1939).
6. Maisel, D. B., and T. K. Sherwood, *Chem. Eng. Progr.*, **46**, 131 (1951).
7. Jacob, Max, and W. M. Dow, *Trans. Am. Soc. Mech. Engrs.*, **68**, 123 (1946).
8. Tessin, William, and Max Jacob, *ibid.*, **75**, 473 (1953).